

# A Model for Improving the Quality of Student Talk in Mathematics Classrooms

Dr. Jeff Zwiers



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## ABOUT THE AUTHOR



Dr. Jeff Zwiers is a senior researcher at Stanford University. He supports the Understanding Language Initiative and collaborates with teachers, instructional coaches, and school systems to improve the education of academic English learners. He co-directs the Academic Language Development Network, which focuses on accelerating students' literacy, language, cognition, and conversation skills. He has also written books and articles on these topics. His current work, supported by a National Professional Development Grant, focuses on developing teachers' practices for fostering students' academic language and literacy across disciplines.

## BACKGROUND

In the last decade, interest has grown in increasing the quantity and quality of student talk in mathematics classrooms. Teachers and researchers have found that when students engage in mathematical talk, they (a) use more mathematical reasoning, (b) build up mathematical understandings, (c) more-effectively solve and craft mathematical problems, and (d) develop mathematical language to enable a, b, and c (Chapin, O'Connor, & Anderson, 2009).

For the purposes of this paper, “mathematical talk” means talk between students, including whole-class, small-group, and paired activities. It includes both structured oral interactions (e.g., pair-shares, answering questions, etc.) and mathematical conversations in which students engage in less structured exchanges using multiple turns.

In this paper, I'll examine the challenges presented by math instruction that mainly emphasizes solving problems for points and grades. Next, I'll outline an effective model for creating a classroom that is teeming with rich mathematical talk focused on building up and communicating important mathematical ideas. We will then look at how to integrate math practices and processes, use math problems effectively, and support these increasingly-rich conversations among students. Finally, I'll introduce the next steps that you can take to bring this to life in your context.

## CHALLENGES

Improving student-student math discourse requires more than just inserting a few new routines into your lessons, or telling students to solve problems together and use the sentence frames on the wall. It requires restructuring of how students think about math: from thinking about problems as an endless series of tasks for points to thinking about problems as building blocks for key mathematical ideas. It requires us to move from a solving mindset to a building mindset, which results in higher quality math talk and deeper learning. Despite the effort it takes, this work is well worth it in terms of student understandings and outcomes (Rabel & Wooldridge, 2013).

A significant challenge is that the common “traditional” model of math learning doesn’t support or encourage rich talk. The traditional model often consists of the teacher showing students how to solve several problems at the beginning of the lesson, then having them solve problems with partners or individually until the bell rings. The goal is to get as many problems correct as possible.

Recently, I asked a fifth grader what it means to learn math. She said, “First, the teacher shows you how to solve problems. Then you solve them with partners. My partner is good at math, so I copy from her. Then you solve them by yourself. If you don’t finish them, it’s homework. It’s all practicing to get the problems right on tests and quizzes.”

In such a model, there are few opportunities for students to engage in the rich language of math that results from high quality talk. During whole-class modeling, which might last half a lesson or more, the teacher does most of the talking. Students might answer with one number, one word, or one sentence. Then in partner work, where the most potential is for talk, the focus is on getting to the right answer. So, in typical pair or small-group solving, one student might show or tell partner(s) how to get to the right answer, often as quickly as possible so that they can move on to the next problem. Usually the students who listen and/or copy tend not to articulate their thinking. And when students are solving problems individually, there is very little math talk at all.

Of course, for each lesson and each student there is a spectrum between no talk and great talk, which can vary depending on the day. This paper will help teachers create conditions that help *each* student move *toward* great math talk, more often.

**“Improving student-student math discourse... requires us to move from a solving mindset to a building mindset, which results in higher quality math talk and deeper learning”**



# HIGH-QUALITY MATH TALK

Fortunately, in recent years many professional development efforts have emphasized talk-based routines that increase the *quantity* of talk in math lessons. This is a good start. But many of these routines focus on solving problems, and even though productive talk can happen in such interactions, we must not limit students' mathematical talk to just problem-solving. Problem-solving is vital, of course, but we need to add structures and supports to make the journeys to solutions—and beyond—much more valuable than the solutions themselves. We want to do all that we can to improve the quality of students' interactions and conversations about the problems, solutions, and ideas that emerge from them (Batista & Chapin, 2019).

## Benefits of Rich Math Talk

Talk can be loud, messy, and even off-task at times, so what makes it worth it—especially for multilingual learners?

### Students...

- Feel a sense of agency and self-efficacy in building up their own ideas and doing what mathematicians do.
- Try out math language on others, create new ways of using language in math, and hear different words and ways of expressing math ideas from partners.
- Get to know others, building relationships and feeling more of a sense of belonging.
- Learn the deeper ideas of how math works; they see patterns and connections.
- Get better at solving problems in math and in other disciplines.
- Learn to collaborate with others.
- Become more engaged in lessons.

### What other benefits can you envision?

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

Below, *Figure 1* provides a model for improving the quality of talk. In the top oval, you will see that the purpose of talking is to build up one or more mathematical ideas, understandings, and arguments. Solving problems, in the lower-left oval, is just one component that students use to build up mathematical ideas through talking. Integrating mathematical practices is the other key component. This process is glued together by use of conversation supports and skills, which includes the work that teachers do to prepare, prompt, model, and scaffold student conversations. The focuses in these ovals can and will vary in emphasis over a single conversation. But usually, if you only focus on one component (e.g., solving problems), conversations are less likely to be extended, productive, and engaging.

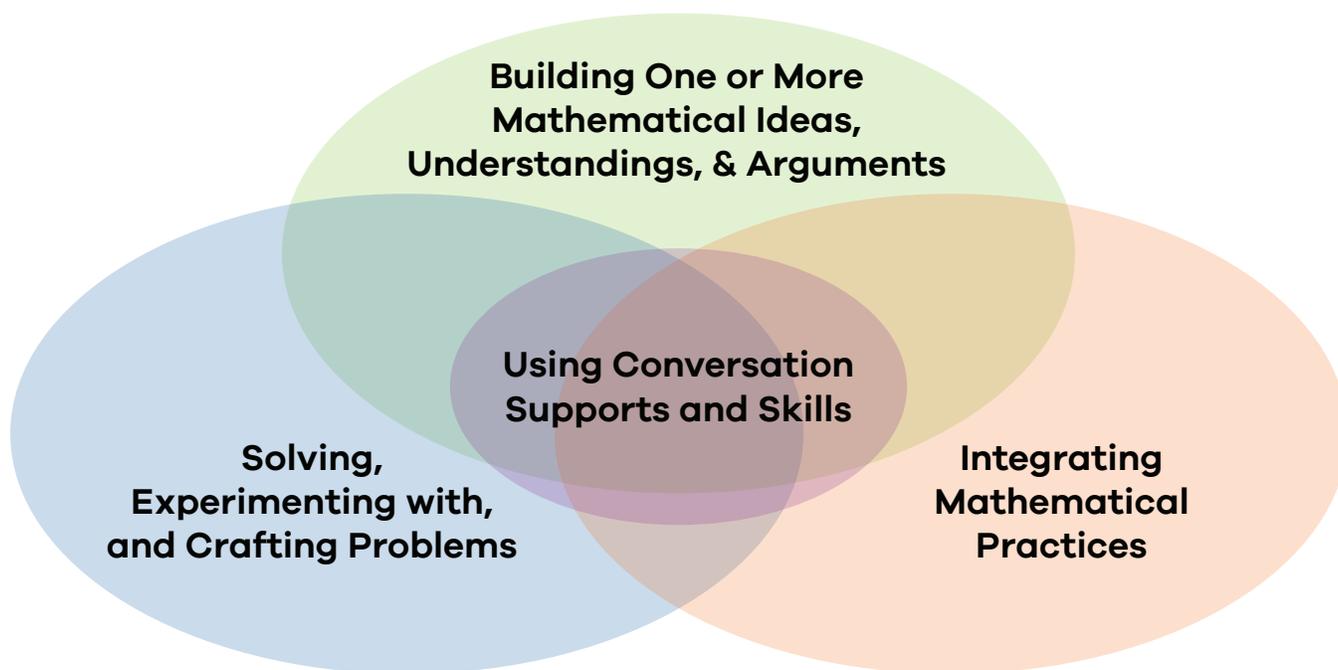


Figure 1. A model for improving the quality of student-student talk in math



## Building Mathematical Ideas, Understandings, & Arguments

A challenge we face is that we tell students to talk, without giving them enough of a reason to talk. If they think that solving problems is the be-all-end-all of math, and they quickly find the answer—or let someone else quickly find the answer—then there isn't much reason to talk. Talking about math is a lot of work, and if there isn't enough motivation or guidance, then students trade a few one-liners to make their teacher happy, and that's it.

Now, if we give students something to build, and they know that their partners can help them build it through conversation, then more and better talk happens. The catch? It requires an idea-building mindset in which students are always building up mathematical ideas. Whenever students solve problems, craft their own problems, create mathematical models, show their work, or talk with peers, they do so to build up ideas about how math works.

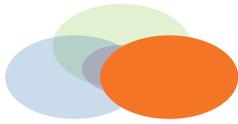
So, how can we enable this idea-building? As you can see at the top of Figure 1, the guiding purpose—or goal—of a conversation should be to develop students' mathematical concepts, claims, and understandings. These big ideas can come from the teacher, students, the curriculum, and math standards. Here are some sample ideas that conversations might build.

- A fraction tells you how many parts of a whole there are.
- Division is how many times one number fits into another.
- When you multiply two fractions, the answer is smaller than both of them.
- To figure out the area of complex shapes, we can break them down into shapes we know.
- I can look at a line on a graph and come up with an equation to tell me all the points on the line.
- I can solve systems of equations different ways such as substitution or “subtraction”
- By looking at the quadratic expression, I can decide whether to factor, complete the square, or use the formula.

There are many other big ideas like these in your curriculum and standards, but often you will need to hunt for them and reword them to make them more interesting and buildable for students.

**“If we give students something to build, and they know that their partners can help them build it through conversation, then more and better talk happens.”**





## Integrating Mathematical Practices and Processes

Mathematical practices and processes include major skills and proficiencies that are needed for learning, doing, and reasoning about math. The practices are necessary for building up ideas, conversations, and solving problems. Included in *Figure 2* below are practices and processes that tend to be most-useful for helping students build up mathematical concepts, together, in conversations. These are based on classroom observations, conversations with teachers, the research, and analyses of a wide range of math standards.

| Practices and Processes  | Sample Student Responses  | Sample Prompts  |
|--|---|---|
| Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate (TEKS Mathematical Process Standards). Notes: Representations are often used to justify and show relationships that are impossible to communicate with just words in sentences.   | <ul style="list-style-type: none"> <li>• I put them into a graph to show how the answer is where the two lines cross.</li> <li>• I drew the pencils in different groups to help me times them up.</li> </ul>  | How would you clearly show others your thinking for solving this problem? Come up with at least two different representations and compare them. |
| Make claims, conjectures, and generalizations, with justification. (CCSS Mathematical Practice Standards). Notes: Conjecture tends to mean coming up with a tentative and logical idea about how things work in math. which includes seeing and using patterns and relationships and explaining how they might be used. Justification of these things means supporting an idea with math principles, patterns, and what is given in a problem. | <ul style="list-style-type: none"> <li>• I think when you multiply two fractions, the answer will always be smaller because...</li> <li>• You can never divide by zero because...</li> <li>• It says in the problem that the submarine ascended.</li> </ul> | In looking at this pattern, can you come up with a claim for how math works? What problems support your idea? Are there counter-examples?       |
| Engage in discussions that reflect on the mathematical thinking of self and others. (Florida B.E.S.T. Standards). Notes: This includes developing students' abilities to construct arguments, justify methods, and compare their responses to the responses of their peers.  | <ul style="list-style-type: none"> <li>• I don't think you can say it's 'always true' based on trying it with just those numbers. It could be another number you didn't try.</li> </ul>   | After listening to a partner's reasoning, what seems right, wrong, or unclear? Why?   |

Figure 2. Mathematical Practices & Processes that Strengthen Conversations

Notice that all three practices are supported by justification. Justification means giving a mathematically logical reason for doing or saying something. Now, consider all the rich language that is needed to talk about these practices. Finally, remember that these can overlap and intertwine in the very same response by a student. A student might start talking about procedures for solving a problem and then come up with a conjecture about what is happening and a representation of the student's evolving idea for how math works.



## Solving, Experimenting with, and Crafting Problems

Most high-quality conversations benefit from doing something with math problems such as **solving**, **experimenting** with, or **crafting** them. Problems can motivate students to use mathematical practices to find the answers. Problems can also help students see patterns and apply knowledge. Problems are very helpful in building up mathematical ideas because they are more concrete examples that students can use as “building blocks” for their ideas.

### Solving Problems

I have noticed a problem with “math problems”: we are not getting all of the math learning that we can out of them. As we saw in the beginning of this paper, many students see math learning as solving an endless array of increasingly challenging problems, for points. They tend to think that math is being done to them: feeling like they have little control over how they think. It doesn't have to be this way. In fact, the model in this paper is a powerful way to turn problem solving into rich mathematical learning, by building up mathematical ideas, rather than just stepping stones to better grades.

With some strategic reframing, we can get students to learn a lot *more* math from problem-solving. There are a variety of problem-solving strategies that can help students solve problems and understand how math works. Some of these strategies are in the first column of *Figure 2* above. Notice the breadth and depth of thinking that can happen in each problem—that is, if we give students time and we don't encourage mindless shortcuts. In conversations with other students, students can “try out” different ways to articulate their thoughts using mathematical language\* while also hearing how other students articulate and communicate their ideas.

\*Mathematical language, while it does include math content vocabulary (reciprocal, denominator, derivative) and dual-meaning words (roots, odd, power, product), also includes any language or combination of cues (movement, visual, etc.) to communicate a mathematical idea.

- Analyzing the problem to see what is happening, to visualize it, or to draw it in some way
- Analyzing the problem for data given, and how the data relate or change.
- Assuming that the problem-writer included enough information to solve it (and doesn't need the solver to add or change anything)
- Estimating and checking progress against the estimate(s)
- Staying focused on what the problem asks you to find
- Planning more than one solution pathway or method
- Using background knowledge of similar problems or situations
- Starting over if you get stuck or end up with an illogical answer

**Example:** Try to solve this problem and keep track of any strategies from the list above that you use along the way.

*It takes Brianna 8 hours to plant an acre full of trees; it takes Alex 12 hours to plant an acre full of trees. How long will it take to plant trees on one acre if they work together?*

If you tried to solve this problem without knowing the algorithm, you likely used several thinking skills such as estimating (It should take less than 8 hours), visualizing and representing (drawing the acre, a timeline, and the amount of work being done), and organizing the units (hours, acre). And if you used the algorithm for rate problems like this one ( $1/r_1 + 1/r_2 = 1/t$ ), I would then ask you to talk with a partner to explain how and why the algorithm works, along with how it might be useful out in the world with other combined-rate problems.

As your students solve problems, they often need to be reminded that the right answer to a problem is not nearly as important as the math that needed to happen to get that answer. Make sure they know that the problems and their solutions will become part of their idea-building and conversations with peers.

## Experimenting with Problems

Students should also experiment with problems. Experimenting with math problems, as in science, means changing certain conditions or constraints to gain some insight or confirm a hypothesis. This means changing the values, and even the conditions, to see what happens. It includes asking lots of “If we changed this, what would happen?” types of questions. It can also include changing the numbers to something easier to work with. For example, you might change Alex's rate in the problem above to 8 hours as well and see what happens, knowing that it will take four hours now. You can also experiment with different representations of the problems such as drawings, charts, and graphs, and compare them.

As you can imagine, when students start experimenting in different ways, they have different strategies and insights to share with others in conversation. That is, they have more information gaps to fill with talking and listening. If, for example, you tell students to solve a problem the fastest way and then have students pair up to share what they did, you won't get much talking or listening.

## Crafting Problems

One of the most powerful learning experiences in math is crafting problems. Unfortunately, those who seem to learn the most from this process are curriculum writers and teachers. Students must be allowed to join in on this learning! We must encourage them to come up with problems that show and support the mathematical ideas they are building. In creating a problem, students learn how to be precise with their words, include enough information for solvers to work with, and think about the problem from a perspective of “will this will help others learn?” rather than “I need to do this problem for points.”

For example, you can ask students to create a business plan that they would pitch to investors. They can come up with an idea for goods or services to sell, and make forecasts based on calculations. A teacher can give parameters related to the math they have been learning, such as wanting to see use of multiplication, division, percentages, or ratios. The teacher reminds students to use conversation skills and math idea building skills as they talk about the plan.

Crafting their own problems also fosters students' feelings of agency. This is especially true when teachers ask students if they can use the students' problems for quizzes, tests, and lessons.

And when students, individually or otherwise, come up with new and engaging problems, there are information gaps to be filled, which means more language is authentically used between students. I also highly recommend having pairs of students converse to co-craft problems. The negotiation of meanings that usually ensues is amazing to watch.



## Using Conversation Supports & Skills

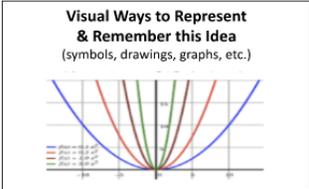
Most math teachers don't spend enough classroom time teaching students how to have these conversations. This must change. As you just read, there are a wide variety of very helpful buildable ideas and types of talk that students can benefit from: mathematically, socially, and linguistically. Using conversation supports and skills can make a significant positive difference.

Conversation supports include activities and routines used before conversations. They help students practice skills and build up ideas that they will co-construct. One routine is called *Stronger & Clearer*: Students interact, sequentially, with more than one peer on the same or

similar topic, so they have the opportunity to make their responses stronger and clearer each time. One application of the routine is having facing lines of students take turns describing to one-another their developing ideas about how math works and sharing problems and solution methods. One line moves to change partners for second and third pair-shares—with an emphasis on making their ideas stronger and clearer each time they share. They also practice face-to-face conversation skills such as valuing others’ ideas, asking questions to clarify and support, maintaining good eye contact, and using connected sentences.

Supports can also include graphic organizers, such as the Math Idea Visual in *Figure 3*. Notice that the information can and should differ across students so they can share what information they have with each other, filling information gaps. The visual puts the idea at the top, and includes various “building blocks” composed of mathematical principles and problems. It is a helpful way to show students that problems are not just disconnected items to be tested on. They are ways to see how math works.

**Idea Building Blueprint - Math**

|  |   |
|--|---|
| <p><b>IDEA</b> (Key Concept; Standard; Answer to Essential Question) <b>in my own words:</b><br/> <b>You can predict what a graph will look like by analyzing the function</b></p> <p><i>Why it's important:</i><br/> <b>Graphs show relationships</b></p> <p><i>How this connects to or depends on other ideas:</i><br/> <b>Domain and range</b></p> <p><i>Real world applications (if any):</i><br/> <b>Bacteria growth</b></p> <div style="text-align: right; border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p style="text-align: center; font-size: small;">Visual Ways to Represent<br/>&amp; Remember this Idea<br/>(symbols, drawings, graphs, etc.)</p>  </div> |   |
| <p><b>Principle, Property, Theorem, Definition</b><br/> <b>Lowest or highest limit of a parabola is a vertex</b></p>   | <p><b>Principle, Property, Theorem, Definition</b></p>  |
| <p><b>Sample Problem</b><br/> <input type="checkbox"/> Given to me <input type="checkbox"/> Found by me <input checked="" type="checkbox"/> Created by me/us<br/> <b>Barrel over a waterfall</b></p> <p><i>How this problem supports/shows the big idea:</i><br/> <b>Negative quadratic parabola - just down</b></p>   | <p><b>Sample Problem</b><br/> <input checked="" type="checkbox"/> Given to me <input type="checkbox"/> Found by me <input type="checkbox"/> Created by me/us<br/> <b>Dolphin jumping</b></p> <p><i>How this problem supports/shows the big idea:</i><br/> <b>Negative quadratic parabola, up and down</b></p> |
| <p><b>Sample Problem</b><br/> <input type="checkbox"/> Given to me <input checked="" type="checkbox"/> Found by me <input type="checkbox"/> Created by me/us<br/> <b>Bacteria growth</b></p> <p><i>How this problem supports/shows the big idea:</i><br/> <b>Positive quadratic parabola</b></p>   | <p><b>Sample Problem</b><br/> <input checked="" type="checkbox"/> Given to me <input type="checkbox"/> Found by me <input type="checkbox"/> Created by me/us<br/> <b>Mt. Everest growth</b></p> <p><i>How this problem supports/shows the big idea:</i><br/> <b>Same ratio over time makes a line</b></p>     |

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Figure 3. Math Idea Building Visual

The setup for a conversation is another support, and the most important aspect of enabling high-quality interactions. Conversation prompts, ideally, should include all four of the dimensions in *Figure 1*, which include asking students to build up a mathematical understanding or claim, doing something with problems, engaging in math practices, and using conversation supports or skills.

Here is a sample prompt focused on building up the idea of ratio, color-coded with the dimensions from *Figure 1*:

### Example Prompt:

***Individually, you and your partner will each solve Problem A or Problem B. Please try to find two different solution methods for solving it. If you can't, don't worry. You might get ideas from your partner. When you are ready to talk, summarize your problem and justify and compare your solution method(s). Your partner should ask you to clarify and justify questions, such as "What do you mean by...?" and "Why did you do that?" Then work together to come up with a clear and strong definition of what a ratio is, using problems as examples. Work together to come up with a new problem that supports your idea.***



True, this prompt is longer than most, but time spent on its crafting is well worth the conversational results.

Now look at the following conversation in a 7th grade math class. See if you agree with the color-coding. The prompt is: ***“Look for patterns in the problems you have been working with during the last three days, come up with your own problem, and use it all to build up a big mathematical idea about how this math works. Ask clarify and justify questions to help you and your partner build up the idea.”***

A: OK, we got it. It's 35 books.

B: It's the answer, but we gotta do more.

A: What do you mean by 'more'?

B: Talk about math, like how this math works. An idea.

A: OK, I got an idea. So this problem you gotta find out what to choose.

B: And when.

A: Yeah. But some other ones like these, you don't choose. It's where they meet.

B: Can you give an example for that?

A: Like the police chase one.

B: But on both kinds siempre se cruzan las líneas on the graph. It's when you change, choose, like the books and apple problems.

A: What do you mean?

B: Like on here, the books graph. If you buy menos que 35 books, we choose this store. But if you buy más que 35 books, this store's better. Costs less.

A: Yeah, or if they're moving, like the bird or police problem, one catches the other where the lines cross.

B: So, we also gotta make a problem, right? How about one about a bike catching a skateboard? Like the skateboard person stole something.

A: I like that.

B: They gotta start at different spots.

A: Why?

B: They don't meet if they start at the same place. And when it starts at more, higher up, its angle is less. What's the word?

A: You mean slope?

B: Yeah. So maybe the skateboard person starts more down the road and, but...

A: But goes slower? So the bike can catch it. And the bike starts up here and goes faster.

B: So, what's our math idea?

A: Maybe it's when two things are changing, you make equations for them, and put them equal and get the answer.

B: And they start different spots, and have different slopes.



- Key:
- Idea Building
  - Using Problems
  - Math Practices & Processes
  - Conversation Supports & Skills

Notice that the students built up the idea that setting different rate equations equal to each other helps to find where they meet, which is the answer. They used the problems they had done to help them craft their own problem and to construct their math idea. They learned from one-another, pushed each other to clarify, and engaged in respectful back-and-forth discourse.

Students can often benefit from extra language support in their math conversations. Remind them that clarity is one of our goals, and that you are providing language that can help clarify their thoughts.

Discourse extenders are also very common supports for language and thinking. They can take the form of prompts, sentence starters, or sentence frames. But make sure students know that they are optional and they are there to help them clarify their great ideas to themselves and for others. Some discourse extenders include:

- What do you mean by...?
- Can you give an example of ...?
- What idea about how math works do these problems help us build?
- What's another way to solve this problem?
- How can we draw what is happening?
- How can we communicate our idea to others?
- Is that always, sometimes, or never true?
- What problem can we come up with that shows our idea?
- Why?
- What will happen if we change the \_\_\_?



## NEXT STEPS

Here are some suggested next steps for fostering the four components of the model for improving student talk in math (in *Figure 1*). These components are color-coded.



Key:

- Idea Building
- Using Problems
- Math Practices & Processes
- Conversation Supports & Skills

### 1. Build linguistic and mathematical confidence for building and sharing ideas

As students build up confidence in their understandings of math and how to express them, they tend to share more. Such an environment requires more than a few community-building activities the first two weeks of school. Fostering a safe environment and culture, where students confidently voice their ideas and arguments, is ongoing. It entails close observation of student interactions, modeling how to respond to others, and building trusting teacher-student and student-student relationships. Students must be reminded that we are all different, we think and say things in different ways, and we learn more as a result.

Confidence also stems from nurturing students' attitudes toward learning math. One of the most important attitudes is knowing and feeling that one can overcome the challenges of learning math *and* build up their own mathematical ideas. We have all had many capable students who didn't believe they were capable. Other positive attitudes include seeing that math is useful, worthy of the work it takes to learn it, and good for the brain. And it is vital for us to help students see the value of conversing with peers in math lessons. Some students don't

### Applying Ellevation Math

One way to foster student confidence for sharing math ideas and taking risks with language is by previewing and front-loading key vocabulary and concepts. Ellevation Math "Primers," for example, interactively teach students new vocabulary and help them see it applied in solving problems. By giving students this individual time to preview and prepare, they tend to feel more confident sharing their ideas with new language in class. For example, a reticent-to-talk student, when participating in a lesson on the Pythagorean Theorem, raised his hand to say "I know that's the *hypotenuse* because it's like the ladder they used in the Primer!"



## 2. Strengthen all learning with mathematical practices and processes

Mathematical practices and processes play vital roles in solving problems, building ideas, and engaging in productive math conversations. And yet, they are often neglected because of the focus on getting answers to problems as quickly as possible. So, as you design lessons and think about how students will use problems to build up ideas, weave in, highlight, model, and emphasize the power of these underlying practices and processes. They are what tend to separate the students and adults who can think mathematically versus those who can just find the answers to problems. For example, a teacher starts with standards focused on dividing fractions. She then weaves in sample solution methods (some incorrect) that students need to argue for or against, as well as explain their reasoning in pairs and small groups.

### Applying Ellevation Math

In Ellevation Math, Primers feature characters modeling problem-solving, often using multiple methods and mathematical thinking. Characters collaborate, question, wonder, decide, and persevere in tying the problem they are solving to the larger math concepts being learned.



## 3. Start off with structured interactions and conversation preparation activities

Such activities include pair-shares, jigsaws, and math talks. But make sure that students are pushing themselves and partners to understand and use the problems, not just answer them. And when they interact, they should push one another to clarify and justify their ideas. And observe how students talk and don't talk, listen and don't listen. This will help you adjust how you set up conversations. For example, you might start with writing, to get students zoomed in on the topic (and quiet) before the room bursts into conversation. The writing usually helps students by "warming up" the brain with content and language for use in conversation. Also try idea-level prompts such as "What is subtraction?" "Why do we set two equations equal to each other?" "How can you figure out pi?"

### Applying Ellevation Math

The prompts used in Ellevation Math's Confidence Questions inspire students to put this into action. Students are asked to practice - in writing - informing, explaining, or arguing their ideas to others. The focus is not on getting to a right answer, but instead on making one's thinking visible (and extendable) to selves and others. And by articulating ideas with new language, students tend to feel more confident participating in discussions.

#### Examples of Ellevation Math Confidence Questions

**Inform:** Use your own words. What is a decimal point? Why is it important?

**Explain:** Explain how to find the slope of a line that is perpendicular to  $y = -2x + 5$

**Argue:** Deanna says that every square is a rectangle. Explain why Deanna is correct.

#### 4. Enhance all that you do with authentic communication

There tends to be a lot of communication in math classrooms in which students use the bare minimum amount of language for extrinsic reasons, such as getting points, looking smart, not getting punished, and so on. Lesson activities with authentic communication, on the other hand, have three features: building up one or more ideas (a claim or concept), extra emphasis on clarifying terms and justifying ideas when talking with others, and filling information gaps. By sharing different “building blocks,” such as problems and solution methods, in order to build up concepts and claims, there is variability in responses (i.e., information gaps that need to be filled), making such conversations fertile soil for extended and productive discourse. This also grows a sense that it’s “worth it” to talk and listen.

##### Applying Ellevation Math

Ellevation Math, for example, uses engaging problems to help students build up math ideas while also providing needed language to give students the opportunity to practice, experiment, and communicate how math works. Ellevation Math primers have a wide range of conceptual focuses such as ratios, proportions, quadratic equations, dividing fractions, etc.



#### 5. Formatively assess students’ evolving ideas

Assessment goes well beyond telling students to “show your work” or asking “How did you solve this?” It entails asking how students are putting the many strategies, problems, practices, processes, and thoughts together to construct a key idea about math: an idea that will last them the rest of their lives. For example, a teacher can circulate around the room observing:

- as each student in a pair shares solution methods to their assigned problem (i.e., Partner A solves Problem A);
- as pairs discuss the concept that the problems are teaching them;
- and as they work to craft a new problem that represents the math.

The teacher takes notes on trends across pairs, paying extra attention to the quality of idea-building, use of practices and processes, students’ use of conversation skills to push themselves to make the idea as strong and clear as possible, and their uses of language that are effective, and any that can be improved.

## 6. Have students share their ideas

Come up with ways in which students can share their ideas. You might already have a test or problem-solving performance task in mind to help you assess learning. But I encourage you to add a performance or product that allows students to “defend” their learning of a math idea and communicate it to others authentically (which means not just to you and not just to be assessed). You can encourage them to:

- use problems that they have solved
- use problems that they have created
- name and describe mathematical practices and processes that helped them
- name and describe mathematical principles, patterns, rules, and reasoning
- use visuals, audio, music, video, live experiences, drama, etc. to communicate the idea as strongly and clearly as possible

During a unit students should know that they will communicate their idea through this performance and/or product at the end. Oral interactions and conversations can and should help them prepare for it.

## CONCLUSION

Student-student talk in math tends to flourish when students have sufficient skills, supports, and prompts to go beyond just discussing how to solve problems. Conversations thrive when students talk about how math works. When they converse with partners to co-construct an idea, they use large amounts of language to clarify the concept’s many abstract and complex parts and relationships. They “think above” the problems, using them as building blocks for their ideas. And throughout this, they are supported in their conversations by teachers and their peers, pushing each other to look at ideas and problems from multiple perspectives. Granted, it does require some redesigning of what it means to learn math: to go from solving problems for points to building up mathematical concepts and claims. But the quality of student learning, language, and conversations is well worth it.

## REFERENCES

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- Chapin, S. H., O'Connor, M. C., & Anderson, N. C. (2009). Classroom discussions: Using math talk to help students learn, Grades K-6. Math Solutions.
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# Idea Building Blueprint - Math

**IDEA** (Key Concept; Standard; Answer to Essential Question) **in my own words:**

*Why it's important:*

*How this connects to or depends on other ideas:*

*Real world applications (if any):*

**Visual Ways to Represent  
& Remember this Idea**  
(symbols, drawings, graphs, etc.)

|  |  |
|--|--|
| <p><b>Principle, Property, Theorem, Definition</b></p>   | <p><b>Principle, Property, Theorem, Definition</b></p>   |
| <p style="text-align: center;"><b>Sample Problem</b></p> <p style="text-align: center;">(<input type="checkbox"/> Given to me   <input type="checkbox"/> Found by me   <input type="checkbox"/> Created by me/us)</p> <hr style="width: 80%; margin: 10px auto;"/> <p style="text-align: center;"><i>How this problem supports/shows the big idea:</i></p> | <p style="text-align: center;"><b>Sample Problem</b></p> <p style="text-align: center;">(<input type="checkbox"/> Given to me   <input type="checkbox"/> Found by me   <input type="checkbox"/> Created by me/us)</p> <hr style="width: 80%; margin: 10px auto;"/> <p style="text-align: center;"><i>How this problem supports/shows the big idea:</i></p> |
| <p style="text-align: center;"><b>Sample Problem</b></p> <p style="text-align: center;">(<input type="checkbox"/> Given to me   <input type="checkbox"/> Found by me   <input type="checkbox"/> Created by me/us)</p> <hr style="width: 80%; margin: 10px auto;"/> <p style="text-align: center;"><i>How this problem supports/shows the big idea:</i></p> | <p style="text-align: center;"><b>Sample Problem</b></p> <p style="text-align: center;">(<input type="checkbox"/> Given to me   <input type="checkbox"/> Found by me   <input type="checkbox"/> Created by me/us)</p> <hr style="width: 80%; margin: 10px auto;"/> <p style="text-align: center;"><i>How this problem supports/shows the big idea:</i></p> |